Coordinate geometry

linear equation $y = mx + c$

$m = \frac{y_2 - y_1}{x_2 - x_1} \iff y_2 - y_1 = m \cdot (x_2 - x_1)$

for perpendicular lines: $m_1 \cdot m_2 = -1 \iff m_2 = \frac{-1}{m_1}$

Three-dimensional

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ (distance between two points)

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$ (midpoint)

Vectors

base vectors → i, j, k $\quad \vec{a} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} = 3i + 2j + 5k$

unit vector: vector with length 1

u.v. with same direction as $\vec{a}$: $\frac{\vec{a}}{|\vec{a}|}$

If points A, B, C collinear (= on same line):

$\vec{AB} = k \cdot \vec{BC}$

scalar product (dot product): $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos(\theta) = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\vec{a} \cdot \vec{b} = 0 \iff \vec{a} \perp \vec{b}$

$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \iff \vec{a} \parallel \vec{b}$

$\Rightarrow \cos(\theta) = \frac{|\vec{a} \cdot \vec{b}|}{|\vec{a}| \cdot |\vec{b}|}$ ← modulus, so the angle is acute

line equation

$\vec{r} = \vec{a} + \lambda \cdot \vec{d}$ (direction vector)

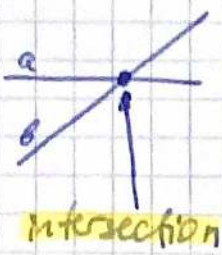
$\begin{cases} x = \lambda \cdot d_1 + c_1 \\ y = \lambda \cdot d_2 + c_2 \\ z = \lambda \cdot d_3 + c_3 \end{cases}$ parametric form

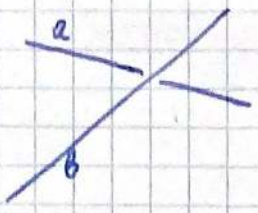
or $c_1 x + c_2 y = c_3 z$ Cartesian form

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$

$\begin{cases} x = \lambda + 3 \\ y = 5\lambda + 2 \\ z = 2\lambda \end{cases}$

$x - 3 = \frac{y - 2}{5} = \frac{z}{2}$





skew lines (=windschief)

parallel lines

Vector product and areas

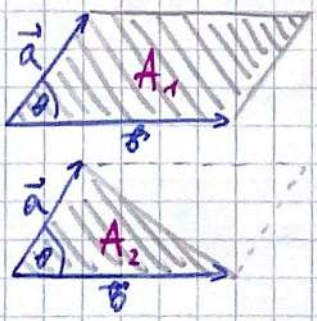
vectors product (cross product): perpendicular vector to both $\vec{a}$ and $\vec{b}$

$$\vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \sin(\theta) = \begin{matrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{matrix}$$

$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$! $\quad \vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ $\quad \vec{a} \times \vec{a} = 0$

$\vec{a} \times \vec{b} = 0 \quad \Leftrightarrow \vec{a} \parallel \vec{b}$

$\vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \Leftrightarrow \vec{a} \perp \vec{b}$



$A_1 = |\vec{a} \times \vec{b}|$

$A_2 = \frac{1}{2} \cdot |\vec{a} \times \vec{b}|$

Plane equation

$$\vec{r} = \vec{a} + \lambda \vec{d}_1 + \mu \vec{d}_2$$

$\vec{a}$: point on plane; $\vec{d}_1, \vec{d}_2$: direction vectors

$$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$ → scalar product form

$\vec{n}$: normal vector → $\vec{n} = d_1 \times d_2$

$$n_1 \cdot x + n_2 \cdot y + n_3 \cdot z = d$$ → Cartesian form