Complex numbers I

$$\underbrace{z}_{\text{complex number}} = \underbrace{x}_{\text{real part}} + \underbrace{i \cdot y}_{\text{imaginary part}}$$ ← cartesian form

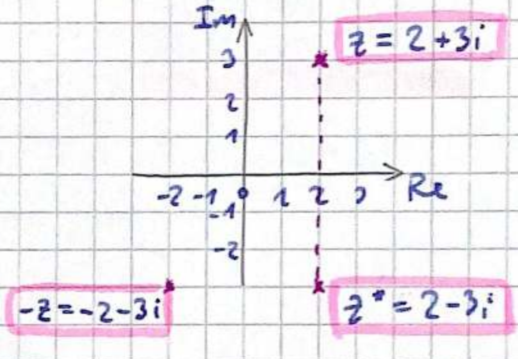
$i = \sqrt{-1}$

$Re(z) = x$ $\quad$ $Im(z) = y$

complex conjugate

$z^* = x - i \cdot y$ $\qquad$ $z \cdot z^* = x^2 + y^2$

complex plane / Argand diagram

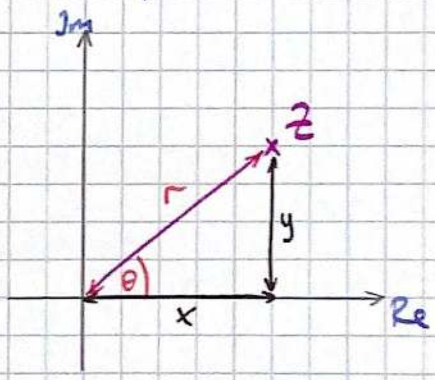


modulus-argument form (polar form):

$r \cdot cis(\theta)$

$r = \sqrt{x^2+y^2}$

$\theta = \arctan\left(\frac{y}{x}\right)$



$z^* = r \cdot cis(-\theta)$

$x = \cos(\theta) \cdot r$

$y = \sin(\theta) \cdot r$

$$r \cdot cis(\theta) = r \cdot (\cos(\theta) + i \cdot \sin(\theta)) = r \cdot e^{i \cdot \theta}$$

↑ Euler form

$$(n \cdot cis(\theta)) \cdot (m \cdot cis(\varphi)) = n \cdot m \cdot cis(\theta + \varphi)$$
$$(n \cdot cis(\theta)) : (m \cdot cis(\varphi)) = \frac{n}{m} \cdot cis(\theta - \varphi)$$

De Moivre's theorem:

$(r \cdot cis(\theta))^n = r^n \cdot cis(n \cdot \theta)$ $\quad$ for $n \in \mathbb{Z}$

Complex numbers II

Roots of $\boxed{\text{polynomials with real coefficents}}$ are either **real** $\boxed{\text{or}}$ occur in **complex conjugate pairs**

$$\boxed{(x-z)\cdot(x-z^*) = x^2 - 2\cdot \mathrm{Re}(z)\cdot x + |z|^2}$$

$$\boxed{\underbrace{1 + \mathrm{cis}\,\frac{2\pi}{n} + \mathrm{cis}\,\frac{4\pi}{n} + \mathrm{cis}\,\frac{6\pi}{n} + \ldots + \mathrm{cis}\,\frac{2\cdot(n-1)\,\pi}{n}}_{n^{th}\text{ roots of } 1 \ (n \text{ solutions})} = 0}$$